M.Sc. Sem II.

MPHYCC - 6

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Magnetosonic Waves

Date

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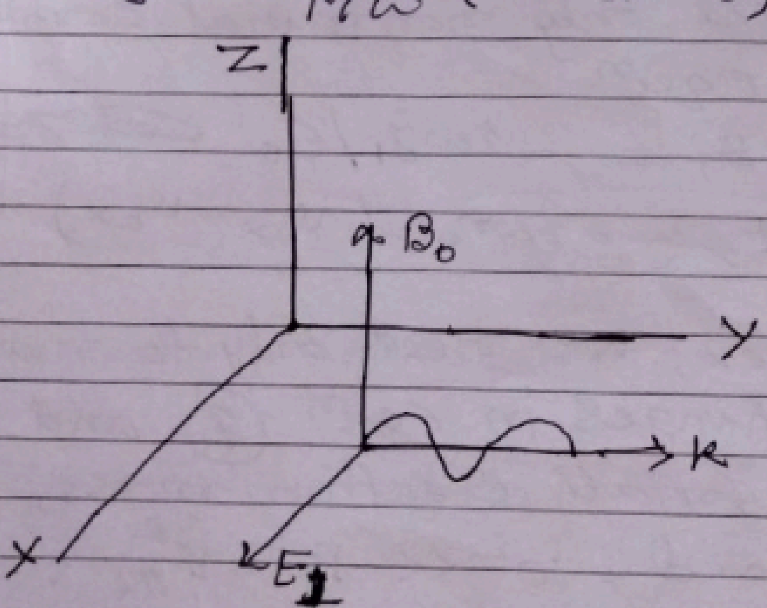
We consider low-frequency electromagnetic waves propagating across B_0. Again we may take $B_0 = B_0\hat{z}$ and $E_1 = E_1\hat{x}$, but we now let $K = K\hat{y}$ (fig). Now we see that the $E_1 \times B_0$ drifts lie along K, so that the plasma will be compressed and released in the course of the oscillation. It is necessary, therefore, to keep the ∇P term in the equation of motion. For the ions, we have

$$Mn_0 \frac{\partial v_{i1}}{\partial t} = en_0 (E_1 + v_{i1} \times B_0) - \gamma_i K T_i \nabla n_1 \quad \text{—} (1)$$

With our choice of E_1 and K, this becomes

$$v_{ix} = \frac{ie}{M\omega}(E_x + v_{iy} B_0) \quad \text{—} (2)$$

$$v_{iy} = \frac{ie}{M\omega}(-v_{ix} B_0) + \frac{k}{\omega}\frac{\gamma_i K T_i}{M}\frac{n_1}{n_0} \quad \text{—} (3)$$



Figure

Geometry of a Magnetosonic Wave Propagating at right angles to B_0.

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The equation of continuity yields

$$\frac{n_1}{n_0} = \frac{k}{\omega} v_{iy} \quad \text{—— (4)}$$

So that equ^n (3) becomes

$$v_{iy} = -\frac{ie}{M\omega} v_{ix} B_0 + \frac{k^2}{\omega^2} \frac{\gamma_i K T_i}{M} v_{iy} \quad \text{—— (5)}$$

with the abbreviation

$$A = \frac{k^2}{\omega^2} \frac{\gamma_i K T_i}{M}$$

This becomes

$$v_{iy}(1-A) = -\frac{i\Omega_c}{\omega} v_{ix} \quad \text{—— (6)}$$

Combining this with equ^n (2), we have

$$v_{ix} = \frac{ie}{M\omega} E_x + \frac{i\Omega_c}{\omega}\left(-\frac{i\Omega_c}{\omega}\right)(1-A)^{-1} v_{ix}$$

$$v_{ix}\left(1 - \frac{\Omega_c^2/\omega^2}{1-A}\right) = \frac{ie}{M\omega} E_x \quad \text{—— (7)}$$

This is the only component of v_{i1} we shall need, since the only nontrivial component of the wave equ^n

$$(\omega^2 - c^2 k^2) E_1 = -i\omega j_1/\epsilon_0 \text{ is}$$

$$\epsilon_0 (\omega^2 - c^2 k^2) E_x = -i\omega n_0 e (v_{ix} - v_{ex}) \quad \text{—— (8)}$$

To obtain v_{ex}, we need only to make the appropriate changes in equ^n (7) and take the limit of small electron mass, so that $\omega^2 \ll \omega_c^2$ and $\omega^2 \ll k^2 v_{th}^2$:

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$$v_{ex} = \frac{ie}{m\omega}\frac{\omega^2}{\omega_c^2}\left(1-\frac{k^2}{\omega^2}\frac{\gamma_e K T_e}{m}\right)E_x \rightarrow$$

$$-\frac{ik^2}{\omega B_0^2}\frac{\gamma_e K T_e}{e}E_x \quad \text{--- (9)}$$

Putting the last three equations together we have —

$$\epsilon_0(\omega^2 - c^2k^2)E_x = -i\omega n_0 e\left[\frac{ie}{M\omega}E_x\left(\frac{1-A}{1-A-(\Omega_c^2/\omega^2)}\right)\right.$$

$$\left. + \frac{ik^2 M}{\omega B_0^2}\frac{\gamma_e K T_e}{eM}E_x\right] \quad \text{--- (10)}$$

we shall again assume $\omega^2 \ll \Omega_c^2$, so that $1-A$ can be neglected relative to Ω_c^2/ω^2. With the help of the definitions of Ω_p and v_A. we have,

$$(\omega^2 - c^2k^2) = -\frac{\Omega_p^2}{\Omega_c^2}\omega^2(1-A) + \frac{k^2c^2}{v_A^2}\frac{\gamma_e K T_e}{M}$$

$$\text{--- (11)}$$

$$\omega^2 - c^2k^2\left(1+\frac{\gamma_e K T_e}{M v_A^2}\right) + \frac{\Omega_p^2}{\Omega_c^2}\left(\omega^2 - k^2\frac{\gamma_i K T_i}{M}\right) = 0$$

Since,

$$\frac{\Omega_p^2}{\Omega_c^2} = \frac{c^2}{v_A^2} \quad \text{--- (12)}$$

equation (11) becomes

$$\omega^2\left(1+\frac{c^2}{v_A^2}\right) = c^2k^2\left(1+\frac{\gamma_e K T_e + \gamma_i K T_i}{M v_A^2}\right) =$$

$$c^2k^2\left(1+\frac{v_s^2}{v_A^2}\right) \quad \text{--- (13)}$$

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where v_s is the acoustic speed. Finally, we have

$$\boxed{\frac{\omega^2}{k^2} = c^2 \frac{v_s^2 + v_A^2}{c^2 + v_A^2}} \quad \text{——— (14)}$$

This is the dispersion for the magnetosonic wave propagating perpendicular to B_0. It is an acoustic wave in which the compressions and rarefactions are produced not by motions along E, but by $E \times B$ drifts across E.

In the limit $B_0 \to 0$, $v_A \to 0$, the magnetosonic wave turns into an ordinary ion acoustic wave. In the limit $KT \to 0$, $v_s \to 0$, the pressure gradient forces vanish, and the wave becomes a modified Alfven wave. The phase velocity of the magnetosonic mode is almost always larger than v_A; for this reason, it is often called simply the "fast" hydromagnetic wave.